2801. Write the parabolae in completed-square form: $y = (x + a)^2 + b$ and $y = (x + c)^2 + d$. Solve for intersections.

2802. Work algebraically: substitute $M = \ln R$ into the curve of best fit and simplify.

2803. This is a quadratic in $x\sqrt{y}$. Factorise and solve for $\sqrt{y}$. You only need deal with the cases at the end: remember that $\sqrt{y}$ must be positive.

2804. In both cases, we are dealing without replacement. So, the numerators and denominators will change as we move along the (imagined) tree diagram.

(a) Multiply probabilities.

(b) Multiply probabilities and ${}^4C_2$.

2805. Consider a discontinuous function.

2806. (a) Find the x intercept in terms of a. Then, using this as a limit, equate one definite integral to the negative of another.

(b) Use your value of a.

(c) Remember that definite integrals calculate the signed area, not the area.

2807. Set the first derivative to zero for SPs. Find the second derivative at these points.

2808. The set-minus notation $A \setminus B$ means the same as $A \cap B'$: elements of A minus those of B. A Venn diagram could help for visualisation.

2809. To prove "necessary", begin with an AP, and show that the fact about the means is true. To prove "not sufficient", find a counterexample, i.e. a non-arithmetic sequence for which the fact about the means is true.

2810. This comes down to showing that the boundary curve of the inequality crosses the circle.

2811. This is a two-tailed test. In a two-tailed test, the critical region is split between the two tails.

2812. Take the LHS, and write its base as e^k. Simplify and use the fact that the resulting expression is symmetrical in the variables a and b.

2813. (a) Reflect the negative values of $f(x)$ in the x axis.

(b) Include extra negative value of y wherever there are positive values of y in the original graph.

(c) This graph consists of the graphs in (a) and (b) superimposed.

2814. Set up the equation $m_1 m_2 = -1$, and show that it has no real roots.

2815. The population is small, so you need to consider binomial probabilities *without replacement*. Find the probability that all five of the sample values are below the upper quartile, and subtract your number from 1.

2816. (a) Rearrange each equation with the trig function as the subject, and use the first Pythagorean identity.

(b) Find $\dot{x}$ and $\dot{y}$. Consider the ranges of these. You don't need to find explicit values of t: all that matters are the respective values of $\sin t$ and $\cos t$.

2817. Translate into algebra, and solve.

2818. Only one of these is true.

2819. Complete the square on the first parabola.

2820. Consider the possibility space as a unit square with vertices at $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$.

2821. The index is $\frac{3}{4}$, which requires calculation of a fourth root. This is only well defined for non-negative inputs. Hence, consider the range of $6\cos x + 7\sin x - 9$. Note that the amplitude of $a\cos x + b\sin x$ is $\sqrt{a^2 + b^2}$.

2822. (a) Your force diagram should contain four forces: reaction, friction, tension and weight.

(b) The triangle in the question is isosceles. Split it in two (on your force diagram) by connecting A to M, the midpoint of BP. Let $\angle BAP = 2\theta$, so that $\angle AMP = \theta$ and the angle of inclination of the rod is $90° - 2\theta$.

Then take moments around A. The reaction and the friction have no moment. Show that the moment of the

- tension is $T\cos\theta$, and of
- the weight is $mg \times \frac{1}{2}\cos(90° - 2\theta)$.

From here, use a couple of trig identities to simplify, and equate the moments.

2823. The radii can be pictured as $1, 2, 3, \ldots$, which is an example of such an AP.

2824. Differentiate the formula $l = r\theta$ wrt time.

2825. Replace one of the inputs with its negative.

2826. Rearrange in order to use the third Pythagorean trig identity.

2827. Find the half-interior angle of the n-gon in terms of n. Then draw a force diagram for one of the pegs, and resolve towards the centre.

2828. Differentiate both sides to set up a DE. Solve this by separation of variables and then integration.

2829. This is not true. Find a counterexample for which $f^2(k) = k$, but for which $f(k) \neq k$.

2830. (a) Sketch such a curve to see what is going on. The quartic $y = (x-1)(x-2)(x-3)^2$ is a simple version.
(b) Consider the first derivative.

2831. Take the factor $\left(1+x+x^2\right)$ out of the sum, leaving a GP to sum.

2832. Place one person, and then their partner.

2833. Find an explicit counterexample: two functions f and g which are not identical, but whose definite integrals are nevertheless equal. The simplest way of doing this is to have functions whose integral is always zero, but which differ at $x = 0$. Note that the functions do not have to be continuous.

2834. Find the equations of the tangent lines at $x = \pm q$. Show that they have the same y intercept.

2835. Without loss of generality, take $\mathbf{p} = \mathbf{i}$ and $\mathbf{q} = k\mathbf{j}$.

2836. Notice that h' has a double root at $x = 1$. Express this algebraically, and integrate.

2837. Begin with the LHS, and multiply top and bottom by $(1 + \sin\theta)$.

2838. The relevant definitions are:
① An even function has $f(-x) \equiv f(x)$, so that $y = f(x)$ has reflective symmetry in the y axis.
② An odd function has $f(-x) \equiv -f(x)$, so that $y = f(x)$ rotational symmetry around O.

2839. Integrate both velocities definitely between $t = 0$ and $t = T$. Equate the displacements, and show that the resulting equation has no roots other than $T = 0$.

2840. Take logs (over base a) of both sides of the first equation. Use a log rule to write this as a linear equation in $\log_a x$ and $\log_a y$. The second equation is quadratic in these variable. Either set $p = \log_a x$ and $q = \log_a y$, or solve directly.

2841. Exactly one equation could generate the graph.

2842. The root itself isn't a nice number, so you can't use it to factorise. Instead, show that the curve $y = 4x^3 + 7x - 8$ has no stationary points, and hence that it is increasing everywhere.

2843. The region in question is a square. Sketch the boundary equations.

2844. One of these is definitely larger than the other.

2845. Solving for intersections yields a non-analytically solvable equation. So, set the first derivative to 2 instead.

2846. Consider the number of rearrangements of seven different letters, divided by an overcounting factor 3!.

2847. (a) The vertices are at the x values which produce mod inputs of 0.
(b) The vertices divide the domain $\mathbb{R}$ into three regions. In each region, use either $|*| = (*)$ or $|*| = -(*)$ for each mod sign, depending on whether its input $*$ is positive or negative.
(c) Just join the dots!

2848. Assume the cube has unit side length. Find the lengths of the relevant diagonals by Pythagoras. Together with a side of length 1, the face and space diagonals form a right-angled triangle.

2849. Quote a result for the derivative of $\cot x$. Then use the third Pythagorean trig identity.

2850. Prove this by contradiction. Firstly assume that neither SP is a point of inflection. Then secondly assume that both SPs are points of inflection. Both cases produce the same contradiction, concerning the gradient of the curve as $x \to \pm\infty$.

2851. Sketch the boundary equations. Consider carefully the location of the vertex of the mod graph, as compared to the parabola.

2852. Consider the behaviour of $\sec\theta$ as $\theta \to 0$.

2853. Neither is true. Find a counterexample to each.

2854. Substitute the latter into the former, and look for a double root.

2855. Find the accelerations a_1, a_2 of the two systems. Then add the magnitudes of these accelerations to find the "acceleration" of the gap size. Use *suvat* on this, with a displacement of 0.5 m.

2856. Simplify with log rules (raise base and input to the same power) before sketching.

2857. (a) The sagitta makes up the outer portion of a radius.

(b) Express the distance from the centre of the circle to the midpoint of the chord as $(r - x)$. Then set up Pythagoras and rearrange to make r the subject.

2858. Use a double-angle formula.

2859. Assume, for a contradiction, that $\text{g}(x)$ has even degree. Show that

① $y = \text{g}(x)$ has at least one SP,

② $y = \text{g}(x)$ has no SPs.

This is a contradiction.

2860. The boundary equation has no real roots. But the inequality is satisfied one side of the x value at which the fraction is undefined.

2861. Use an input transformation: replace y by ...

2862. (a) Use Pythagoras.

(b) Set the squared distance to 18, and solve the resulting boundary equation. Then consider the inequality itself.

2863. The best linear approximation is the tangent.

2864. (a) Consider the factors of the bases.

(b) Use the factor theorem.

2865. Split the fraction up, integrate and simplify.

2866. Assume, for a contradiction, that two of the lines of action intersect at P and the other doesn't pass through P. Consider moments about P.

2867. (a) Evaluate the derivative and use the formula $y - y_1 = m(x - x_1)$.

(b) Take out a squared factor corresponding to the double root at the point of tangency.

2868. This is a quadratic in $\ln x$.

2869. Use the parametric integration formula

$$A = \int_{t_1}^{t_2} y \frac{dx}{dt}\, dt.$$

Solve $y = 0$ to find the limits t_1 and t_2, and set up a definite integral with respect to t.

2870. Assume, for a contradiction, that the polynomial f is periodic, with period T. So, $\text{f}(nT) = \text{f}(0)$ for all $n \in \mathbb{Z}$. Consider the number of roots of the equation $\text{f}(x) = \text{f}(0)$.

2871. Two are true and one is false.

2872. The greatest and least values of n are 9 and 1.

2873. Set up an equation to zero and factorise. Justify the fact that one of the factors does not produce any roots.

2874. Show that the graph has two symmetrical SPs in the correct locations, one max and one min. Also, consider axis intercepts and behaviour as $x \to \pm\infty$. You could also show that the curve has odd symmetry.

2875. Use the quadratic discriminant, noting that one value of m stops the curve being a quadratic at all.

2876. Translate into algebra, starting with $x^2y^3 = a$ for some constant a. Rearrange to make y the subject. Evaluate the derivative at the relevant points.

2877. Evaluate the four end-of-branch probabilities.

2878. Differentiate by the chain rule and simplify. You'll then need to multiply the top and bottom of the resulting fraction by $x - \sqrt{1 + x^2}$: the technique for rationalising the denominator of a surd.

2879. Call the forces A and P_1, P_2, P_3. Then consider the resultant force in the direction of A.

2880. Solve $y = 8$, and find the equation of the tangent line at this point.

2881. There are two ways to get terms in x^7, by selecting

① three instances of x^2, one of x and one of 1,

② two instances of x^2 and three of x.

2882. Only one of these is true.

2883. Enact the differential operator, and rearrange for $\frac{dy}{dx}$.

2884. The implication doesn't go both ways.

2885. Use the first Pythagorean trig identity.

2886. (a) Draw a force diagram for the kite.

(b) Consider the horizontal component of tension, acting on the boy.

(c) Draw a force diagram for the boy.

2887. Choose the chairperson first, then the secretaries, then the adjutants.

2888. Only one of these is true.

2889. This is a quadratic in x^2; factorise it.

2890. There are six ways in which this can happen:

$$\begin{array}{ll} 1,2,3,4,5 & 1,2,4,5,6 \\ 1,2,3,4,6 & 1,3,4,5,6 \\ 1,2,3,5,6 & 2,3,4,5,6. \end{array}$$

2891. It is not possible.

2892. Use the factorial definition of ${}^n\mathrm{C}_r$.

2893. Solve each inequalities separately, then describe the union of the solution sets.

2894. Find the t values associated with the endpoints of the region, by solving $x = 0$ and $y = 0$. Then set up the definite integral and find its value.

2895. Sketch the curves, then use the standard method for calculating the area of a segment.

2896. Assume, for a contradiction, that there is such a polynomial function f, for which $\mathrm{f}(x) \equiv e^x$. Then consider $\mathrm{f}'(x)$.

2897. Solve for intersections, and look for double roots in the resulting polynomial.

2898. Use the second Pythagorean trig identity on the second equation.

2899. Find $\frac{dy}{dx}$ in terms of t using the chain rule (also known as the parametric differentiation formula). Then sub into the LHS of the DE and rearrange to the RHS.

2900. Consider the situation graphically, noting that the parabolae are reflections of one another in the line $y = x$. Work out how many intersections there are when $a = 0$ and $a \to \pm$. At the boundaries between behaviours, the curves must be tangent.

——— End of 29th Hundred ———